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Borsa İstanbul Review

Borsa İstanbul Review 22-6 (2022) 1221–1237
http://www.elsevier.com/journals/borsa-istanbul-review/2214-8450

Full Length Article

State-dependent hedge strategy for crude oil spot and futures markets

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Received 3 March 2022; revised 16 August 2022; accepted 16 August 2022
Available online 28 August 2022

Abstract

Relying on the hidden Markov model improved by the particle swarm optimization algorithm (PSO-HMM), we develop a dual-decision method to address the issue of state-dependent futures hedging. Our approach is attractive in two ways. First, it uses the PSO algorithm to overcome the shortcomings of the traditional algorithm, which can easily fall into the local optima to estimate parameters in a hidden Markov mode. Second, this paper proposes a new hedge position adjustment method based on the identified market states, instead of sticking to the hedge position calculated by the commonly used GARCH-type models to achieve a better trade-off between risk hedging and return acquisition. Specifically, we first improve the accuracy of parameter measurement and employ the PSO-HMM to identify two market states, bear and bull, and fully illustrate the rationality and effectiveness of the proposed model. Based on the market states identified, we then adjust the hedge ratio estimated by GARCH-type models and compare the hedging effects of no hedge, model-driven, and state-dependent strategies. Our empirical results show that the PSO-HMM method can improve the accuracy of state identification over the classical HMM. The market state-dependent hedging strategy has better performance than other strategies when it comes to the trade-off between the return and the risk of a hedged portfolio. Furthermore, robustness checks under different conditions confirm that the state-dependent hedging strategy outperforms the model-driven hedging and no hedge strategies. Thus, our research sheds new light on conventional hedging models.

Keywords: Model-driven hedging strategy; PSO-HMM; State-dependent hedging strategy; State identification

1. Introduction

The price of oil is highly volatile. International crude oil prices are affect by related events through various channels of influence. The factors that affect the situation include not only political conflicts in oil-producing areas but also unexpected events, such as terrorist attacks and the discovery of new crude oil deposits. These factors, coupled with widespread media coverage, lead to fluctuations in oil prices that exceed public expectations. At the same time, because international crude oil has characteristics of both commodities and finance, short-term fluctuations in crude oil prices are closely related to speculation (Zhang & Wu, 2019). In recent decades, the price of crude oil has fluctuated sharply. During the energy crisis in the 1970s, the sharp spike in oil prices had a huge impact on the global economy. Oil prices rose dramatically when the Persian Gulf War broke out in 1990. Then, in the Asian financial crisis in 1997–1998, oil-exporting countries increased production, and the price of Brent crude fell rapidly. Similarly, in 2008, after the global financial crisis began, the price of Brent crude oil fell by more than 70 percent in six months. After the emergence of Covid-19 in early 2020, oil prices suffered another large shock. In 2022, crude oil prices quickly rose because of the Russia-Ukraine conflict. Because of this extreme instability, it is important for risk in crude oil prices to be managed effectively.

Futures contracts are excellent instruments for risk management and are frequently used to hedge commodity risks. Futures hedging means hedgers manage price risk by taking a position in the futures markets that is the opposite of cash. The

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Peer review under responsibility of Borsa İstanbul Anonim Şirketi.

https://doi.org/10.1016/j.bir.2022.08.008

fundamental challenge in hedging futures is determining the optimal hedge ratio (OHR), which is the optimal number of holdings of futures necessary to hedge a single spot exposure. Since the early work by Johnson (1960), a great deal of research has emerged about how to determine OHR. For example, minimum-variance hedging (MVH) is the classical technique used to determine OHR. Many more recent studies have discussed the use of the framework of minimum variance for hedging risk (see Basher & Sadorsky, 2016; Chun et al., 2019; Evrim Mandacı & Kırkpınar, 2022; Hachicha et al., 2022; Wang et al., 2014).

Despite its elegance, the traditional method for determining the MVH ratio is impractical, because it ignores the impact of market conditions on hedging decisions. Hedgers can face state-dependent constraints when they decide the final OHR. So, it is not surprising that hedgers make different decisions in bull and bear markets. As stated by Chang et al. (2010), investors change their established hedging methods according to the trends or patterns in price movements, because the hedging efficiency of futures differs between bearish and bullish markets. Thus, hedgers are advised to adjust their hedging strategies in response to the state of the market environment.

At present, Markov-switching generalized autoregressive conditional heteroskedasticity (MS-GARCH) models are commonly used in considering state dependence in hedging oil futures to capture the volatility in oil markets (Chkili, 2017; Dark, 2015; Maciel, 2021; Pan et al., 2014; Yan & Li, 2018). The above-mentioned studies load the states with endogenous switches governed by the Markov process to characterize asset volatility. However, the drive of exogenous market states should not be overlooked. The idea behind hedging using futures contracts is to take a position in futures contracts such that the return on futures trading can offset the return on the spot market. Therefore, in hedging futures, the buyer or seller compensates for the loss of futures (spots) by sacrificing profit and losses on spots (futures). In that case, if hedgers believe that market conditions are favorable, they can appropriately earn income by adjusting the futures position, instead of strictly following rigid model results. In other words, a state-dependent hedging strategy can reduce a hedger's unnecessary losses. The natural questions, then, are how the state of the market environment can be identified and how the hedging strategy should be adjusted to different market states accordingly.

In general, market states cannot be observed directly, which means that whether the current market condition is bearish or bullish is unknowable. Although market states cannot be observed, we can speculate about them based on available indirect information. This process of indirect identification can be modeled with the hidden Markov model (HMM). HMM is a well-known model for regime identification, widely used in engineering and science. In recent years, HMM has become a popular model for predicting the state of financial markets. Nystrup et al. (2017) state that HMM can mimic the financial market's tendency to alter behavior abruptly as well as to maintain new behavior for extended periods following a change based on information observed. Many scholars have argued that asset allocation based on HMMs is profitable for investors (Bae et al., 2014; Guidolin & Timmermann, 2007; Kritzman et al., 2012; Liu & Wang, 2017). Although these studies consider the problem of dynamic asset allocation, HMM is rarely used for hedging futures.

When HMM is used to identify market states, whether in asset allocation or hedging applications, the validity and correctness of the identification are important indicators. The estimation method for the model parameters of HMM is critical because the results of parameter estimation directly affect the accuracy of the model. At present, the most powerful training method for HMM is the Baum-Welch (BW) algorithm (Baum et al., 1970). But as Rasmussen and Krink (2003) state, although the BW algorithm is fast, it encounters the problem of stagnation at local optima because of the hill-climbing algorithm. The BW algorithm heavily depends on the initial values of the model parameters. Inaccurate estimations of the model's initial values can cause the algorithm to fall into a locally extreme value. Fortunately, the particle swarm optimization (PSO) algorithm can perform a global search as a random search algorithm and detect a globally optimal solution (Shen et al., 2019). Moreover, the PSO algorithm has the following advantages over other evolutionary algorithms. First, the PSO algorithm is a derivative-free optimization technique that can handle any type of objective functions (e.g., nonconvex, nondifferentiable, and discontinuous). Second, the PSO algorithm has fewer parameters to tune and does not require a good initial population to search for optimal solutions. Third, PSO can easily be integrated with other optimization algorithms (AlRashidi & El-Hawary, 2009). Because of the superior performance of PSO, some papers incorporate the PSO algorithm to improve the HMM model, increasing the predictive performance of the model, and apply it to speech recognition (Najkar et al., 2010) and bioinformatics (Lalwani et al., 2015; Sun et al., 2012). Therefore, we use the PSO algorithm to estimate the parameters in HMM (hereafter, the PSO-HMM method) to achieve more accurate results.

This paper improves hedging performance by considering the market state. In general, the paper makes the following three contributions. First, in engineering applications, such as audio recognition and picture identification, some research has coupled PSO and HMM. We apply the PSO algorithm to increase the accuracy of market state identification and extend the improved HMM model to financial risk management. Second, the effect of the market state on investors' decisions is fundamental. The identification results of the market state directly affect whether hedgers adopt appropriate strategies. It is unwise to use hedging measures under favorable market conditions because doing so will offset the benefits they should have obtained. Although some scholars have studied the state identification of volatility using regime-switching models, few of them consider the influence of the market state in research on hedging futures. Our research combines the macro market state and hedging strategy. Third, we propose a new hedge position adjustment method based on the market states identified, instead of sticking to the position calculated with the commonly used GARCH-type models to improve the performance of the hedging strategy. The superiority of the new method is embodied in better trade-off risk hedging and return acquisition. Therefore, our model and method are more feasible in practice.

The remainder of this paper is organized as follows. Section 2 introduces the classical HMM and the PSO-HMM. Section 3 presents two kinds of hedge strategies: model-driven hedging and market-state-dependent hedging. Then the market state identification results and hedging effect are presented in Section 4. Section 5 discusses the robustness checks under different market conditions. Section 6 concludes the paper.

2. The model

2.1. Hidden Markov model

The discrete HMM by Rabiner (1989) entails two processes: an unobservable state process $S = \{s_1, s_2, \cdots, s_t\}$ and an observed process $O = \{o_1, o_2, \cdots, o_t\}$, where s_t is the notation of the states, and o_t denotes observations at time t. s_t and o_t are in finite state spaces, as $s_t \in \{q_1, q_2, \cdots, q_N\}$, $o_t \in \{v_1, v_2, \cdots, v_M\}$, where N is the number of the hidden states, and M is the number of different observations per state. The unobservable process is a Markov chain that cannot be observed directly but can be inferred from the underlying set of stochastic processes associated with each state. In addition, the current state of that Markov chain determines the distribution of another observed process at any given moment.

The relationship between the hidden state sequence and the above observation sequence is shown in Fig. 1.

As shown in Rabiner (1989), HMM is characterized by the following parameters:

- $A = a_{ij}$: the hidden state transition probability matrix, where $a_{ij} = P(s_{t+1} = q_j | s_t = q_i)$ represents the probability of the hidden state q_i transferred to q_j at time $t + 1$, and the matrix size is $N \times N$.
- $B = b_j(k)$: the observational state emission probability matrix, where $b_j(k) = P\ (o_t = v_k | s_t = q_j)$ represents the probability that the hidden state q_j produces the observed state v_k, and the matrix size is $M \times N$.
- $\pi = \pi_i$: the initial state probability distribution, where $\pi_i = P\ (s_1 = q_i)$, $1 \le i \le N$. The complete set of parameters of HMM can be represented by a triple of $\lambda = (A, B, \pi)$. And the constraints $\sum_{i=1}^{N} \pi_i = 1$, $a_{ij} \ge 0$, $\sum_{j=1}^{N} a_{ij} = 1$, $b_j(k) \ge 0$ and $\sum_{k=1}^{M} b_j(k) = 1$ hold.

Based on a given observable data set, the parameters of an HMM model can often be estimated using the Expectation-Maximization (EM) algorithm, also known as the BW algorithm. Using the estimated parameters, the Viterbi algorithm (Viterbi, 1967) can decode the most likely state sequence, thus revealing the evolution of hidden states over the sample period.

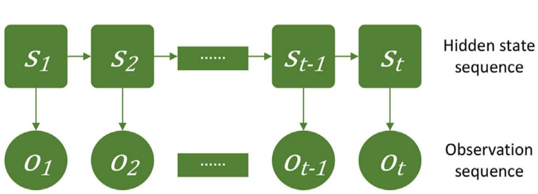


Fig. 1. The structure of HMM.

2.2. Optimized hidden Markov model

When the model parameters are estimated, the traditional BW algorithm can easily get stuck at locally optimal values. In order for the estimation results to be more accurate, the initial parameters of the HMM need to be as close as possible to the global optimal values. As stated by Rabiner (1989), the initial value of B in HMM has the greatest impact on the estimation results. Hence, we use the PSO algorithm to determine the ideal initial value of B. The specific optimization process is as follows.

Step 1 **Initialization:** generate N parameters of B_0 randomly.

Step 2 **Train HMM:** train HMM with B_0 as the initial parameters by BW algorithm.

Step 3 **Fitness criterion:** based on all the HMMs trained in Step 2, we perform the maximum possible state estimation and obtain the corresponding estimation sequence of states. The difference between the state estimation sequence $\hat{S} = \{\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_t\}$ and the reference sequence $S = \{s_1^P, s_2^P, \ldots, s_t^P\}$ is $X = \hat{S} - S$. We choose the sum of the squared elements in X as the benchmark for measuring fitness. Let

$$\delta = \sum_{i=1}^{n} x_i^2 \tag{1}$$

where x_i is the ith value of X. Here, we mark the reference sequence using the method proposed by Pagan and Sossounov (2003).

Step 4 **Update particle position and speed:** adjust the particle's position and speed by comparing the fitness values δ after each update.

Step 5 **Termination conditions:** determine the maximum training number, and set a minimum allowable error value, when the number of training sessions reaches the upper limit, the emission matrix B corresponding to the state estimation is globally optimal.

The general steps of the designed algorithm PSO-HMM can be summarized in Fig. 2 as follows.

3. Hedging strategies

3.1. Futures minimum-variance hedge ratio determination

In the framework of futures hedging, a hedge is usually defined as an investment position in the futures market with the aim of offsetting potential losses that may be incurred by spots. Consider a hedger who holds a long position in a spot and wants to minimize the risk of price fluctuations.

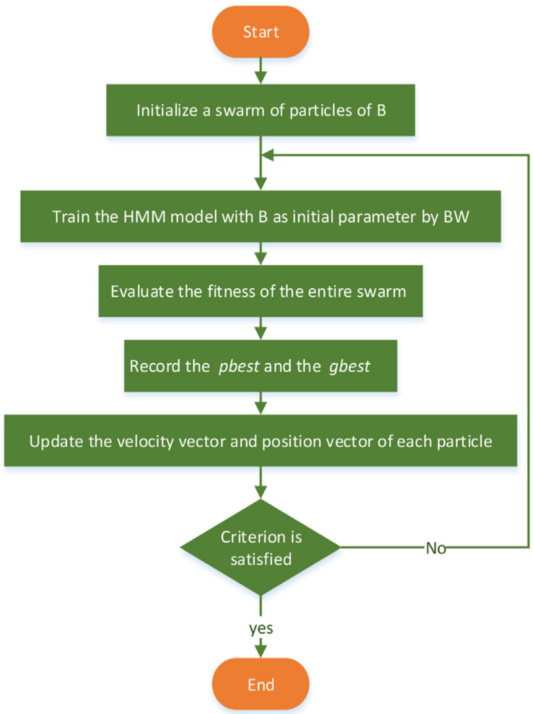


Fig. 2. The PSO-HMM procedure.

The hedged portfolio's return, R_t^{hedge} is denoted by $R_t^{hedge} = R_t^s - h_t \times R_t^f$, where R_t^s and R_t^f are the spot and futures returns at time t, respectively, and h_t is the hedge ratio. According to Johnson (1960), the variance of the hedged portfolio's return is given by

$$var(R_t^{hedge}) = var(R_t^s) - 2h_t cov(R_t^s, R_t^f) + h_t^2 var(R_t^f) \tag{2}$$

where $var\ (\cdot)$ and $cov\ (\cdot)$ are the variance and covariance, respectively. The minimum-variance hedge ratio is the value of h_t that minimizes $var(R_t^{hedge})$.

The OHR is calculated using the first derivative of Eq. (2) with respect to h_t (Baillie & Myers, 1991).

$$\begin{aligned} h_t^* &= \frac{cov(R_t^s, R_t^f)}{var(R_t^f)} \\ &= \rho_t^{s,f} \times \frac{\sqrt{var(R_t^s)}}{\sqrt{var(R_t^f)}} \end{aligned} \tag{3}$$

where h_t^* denotes the OHR and $\rho_t^{s,f}$ denotes the correlation coefficient between spot and futures returns.

3.2. Model-driven hedge strategy

In Eq. (3), the key step in obtaining the hedge ratio is calculating volatility. The model-driven hedge strategy in this paper refers to using volatility models to estimate and predict the volatility of spot and futures returns and then calculating the hedge ratio based on the volatility obtained. Because this paper focuses on state-dependent hedging strategies, without losing generality, the volatility models used in this paper include DCC-GARCH, DCC-IGARCH, DCC-TGARCH, DCC-NAGARCH, and DCC-GJRGARCH. The definitions of the volatility models mentioned above are in Appendix B.

To obtain the dynamic hedge ratios, we split the data into two parts: a training sample and a testing sample, and then performed a one-step-ahead expanding window forecast. The entire sample size is marked as T, and the training set size is marked as w. That is, we use the first w observations as the first estimation window to predict the volatility of period $w + 1$ and estimate the minimum-variance hedge ratio using Eq. (3). We then recalculate the hedge ratio of period $w + 2$ using the data from period 1 to period $w + 1$, and so on. As a result, it produces all dynamic hedging ratios of size $T - w$.

3.3. Market state-dependent hedge strategy

The market state-dependent hedge strategy in this paper refers to adjusting the model-driven strategy based on the market states identified by PSO-HMM. Specifically, we first obtain the hedge ratio based on volatilities calculated with GARCH-type models. Then, by embedding the PSO algorithm

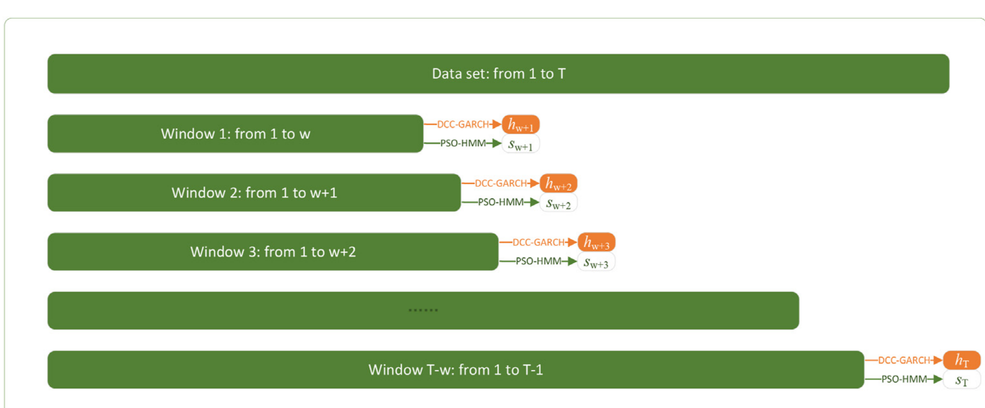


Fig. 3. The state-dependent hedge procedure.

Table 1
Descriptive statistics for Brent crude oil daily returns of spot and futures.

Statistics	Spot	Futures
Observation	2528	2528
Mean	-1.580×10^{-4}	-1.680×10^{-4}
t-Statistics ($\mu = 0$)	−0.271 (0.786)	−0.362 (0.718)
Variance	8.610×10^{-4}	5.460×10^{-4}
Maximum	0.412	0.184
Minimum	−0.644	−0.280
Skewness	−3.229	−1.023
Kurtosis	119.548	20.892
Q (10)	92.711 (0.000)	14.718 (0.143)
Q2 (10)	503.930 (0.000)	478.990 (0.000)
ARCH(10)	50.530 (0.000)	31.970 (0.000)

Note: The Q (10) shows the Ljung-Box statistics of the correlation test for the return series, and $Q^2(10)$ is that for the square of the return series. ARCH(10) are Engle (1982) Lagrange multiplier test statistics for residual ARCH effects at lags 10. The numbers in parenthesis are the *P*-values of the corresponding hypothesis tests.

in the traditional HMM, we use the expanding window to recursively identify whether the market is bullish or bearish. Finally, we adjust the hedge ratio of the model-driven hedge strategy according to the market conditions. The following is the specific adjustment criterion.

- If the last day is in a rising state → Next day: $R_t^{hedge} = R_t^s - \theta_{up} \times h_t \times R_t^f$
- If the last day is in a descending state → Next day: $R_t^{hedge} = R_t^s - \theta_{down} \times h_t \times R_t^f$

where $\theta_{up}(\theta_{up} > 1)$ is the expansion ratio relative to model-driven position h_t, and $\theta_{down}(\theta_{down} < 1)$ is the reduction ratio. Fig. 3 illustrates the procedure for a state-dependent hedge.

4. Empirical results

4.1. Data description

Energy prices have an impact on every sector of the economy, including households, businesses, and most levels of government. Therefore, hedging energy price risk is a critical concern for individuals, businesses, and policy makers (Evrim Mandacı & Kırkpınar, 2022; Shrestha et al., 2017). Fluctuations in the price of crude oil, which accounts for nearly two-thirds of global energy consumption (Wang et al., 2018), has a huge impact on the global economy. In the past, the global oil market has suffered many huge price shocks. As recently as April 2020, price in the crude oil market fell sharply, and the price of WTI crude oil futures was even negative. The overnight shock to the crude oil market hit the participants hard. Large fluctuations in oil prices and market uncertainty require investors to adopt effective hedging strategies to reduce investment risk. Therefore, we look at crude oil futures hedging as an example for empirical analysis.

4.1.1. Descriptive statistics

In the empirical analysis, we use the Brent crude oil spot closing price and Brent crude oil futures settlement price from August 1, 2011, to August 1, 2021. All the data come from the Choice Financial Database. The return R_t is defined as

$$R_t = log(P_t) - log(P_{t-1}) \tag{4}$$

where P_t is the value of the price at time t.

Table 1 shows descriptive statistics for spot and futures series. According to the variances, the spot return fluctuate slightly more than the futures return. In addition, the two return series are leptokurtic and negatively skewed, with kurtosis significantly higher than 3, so it can be inferred that the returns on Brent crude oil have fatter tails than the normal distribution.

The values of the Q statistic show significant serial correlation between the spot and futures returns. Also, the two return series have a nonlinear autocorrelation relationship. In addition, the ARCH effect tests reject the null hypothesis of a correct model specification, indicating strong heteroskedasticity, which suggests that a long-term memory GARCH-type model can be applied in this study later.

4.1.2. Data preparation for the HMM model

Fig. 4 plots the daily spot and futures prices of crude oil from August 1, 2011, to August 1, 2021, showing that the two series are very close to each other and are basically synchronized. We assume that the states of the two markets are consistent. Therefore, we identify the market states based on spot prices, and we believe that the futures market state is the same as the spot market state. In this paper, a set of spot return sequences including daily, five-day, and ten-day returns are used as observations in the HMM. One reason for this consideration is that the sequence of multiple frequencies can

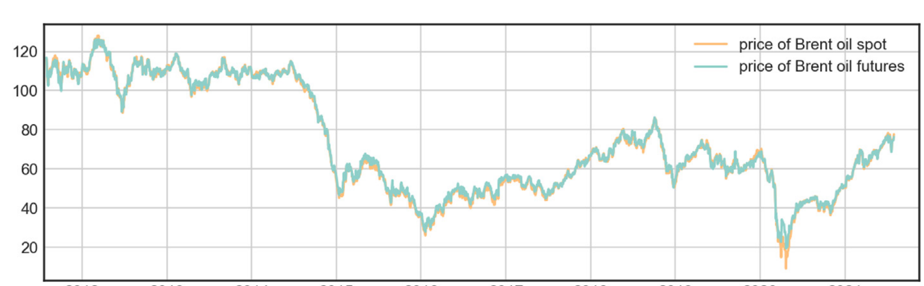


Fig. 4. Daily prices of Brent oil spot and futures.

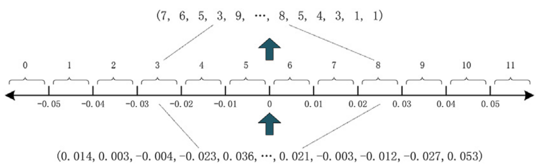


Fig. 5. Coding process of the daily return into a set of symbols.

provide more historical information, which is conducive to accurately identifying the market states for HMM.

4.1.3. Coding data into HMM symbols

When the observation sequence is continuous, the probability $b_j(k)$ that a given hidden state s_t produces a random observation o_t should be expressed as a continuous probability density function, and most methods approximate this relationship with a normal distribution.

According to the descriptive statistics in Table 1, the normal distribution dose not accurately model the probability distribution of the Brent crude oil return series. So, we use the method of discretizing stock returns to convert the probability of the observations generated by the HMM model into a discrete value. Specific steps are described hereafter, and Fig. 5 displays an example of the coding process of the daily return.

Because the return contains outliers, we first select the interval range $[-L, L]$ and then divide the small intervals of equal distance (V) in positive and negative directions, with 0 as the center point, until the cell contains the maximum value L and the minimum value $-L$. And we also set two open intervals greater than L and less than $-L$, respectively. Finally, the yield value is transformed to the relevant discrete value based on the interval in which it is located. Here the values of L of the daily return, five-day return, ten-day return are 0.05, 0.1, and 0.15, and the values of V are 0.01, 0.02, and 0.03, respectively. In Fig. 6, we conclude that the interval settings in this paper are reasonable.

4.2. Market state identification results

Figs. 7 and 8 show the market state identification results decoded by HMM and PSO-HMM, respectively. Fig. 7 shows that HMM can capture the main trends in the crude oil market in the full sample period. The blue scatter plot represents a bear market, and the red scatter plot denotes a bull market. It seems clear that several market states based on HMM are misjudged during some periods (e.g., the period October 2014 to December 2014). Therefore, we are inspired to improve the identification accuracy of the model. The identification results based on the novel model called PSO-HMM in this paper are shown in Fig. 8.

Following Liu and Wang (2017), we present the empirical statistical results of HMM and PSO-HMM in Table 2, then confirm some of our interpretations. The Z-statistic for the two models indicates that the mean of state 0 is substantially below 0 at the 1 percent significance level, whereas the mean of state 1 is significantly above 0 at the 1 percent significance level. In addition to the fact that negative returns are more common than positive returns in state 0, positive returns are also more common than negative returns in state 1. The findings show that hidden states, which can be understood as market conditions, govern the time-varying distribution of returns. Specifically, state 0 is associated with bear markets, whereas state 1 is associated with bull markets. State 0 has a higher standard deviation than state 1, which indicates greater volatility in a bear market, and the Z-statistics and return frequency of PSO-HMM have higher significance. Furthermore, the frequency of positive returns in state 1 identified by PSO-HMM is 0.61922, which is larger than HMM, and the frequency of negative returns in state 0 identified by PSO-HMM is also more than

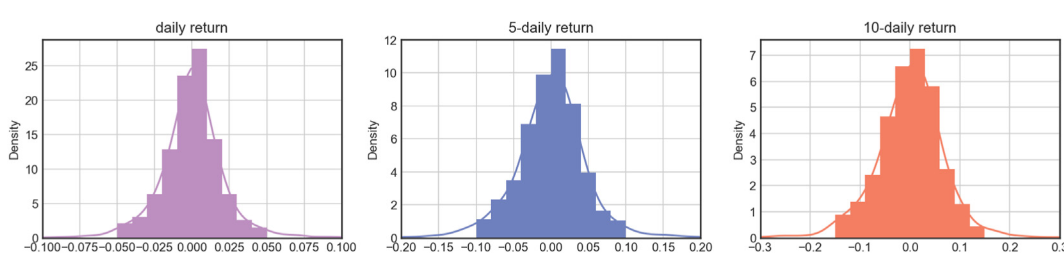


Fig. 6. The distributions of daily return, five-day return, and ten-day return.

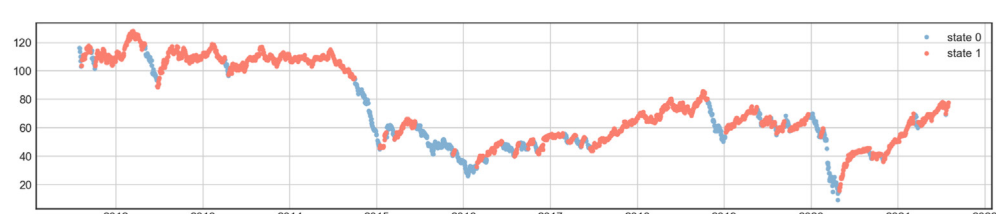


Fig. 7. Market state identification results based on HMM.

Fig. 8. Market state identification results based on PSO-HMM.

Table 2
Comparison results of HMM and PSO-HMM in identifying market states.

	HMM		PSO-HMM	
	state 0	state 1	state 0	state 1
Mean	−0.00822	0.00273	−0.00624	0.00503
Std.	0.04761	0.01808	0.03611	0.02061
Sample size	666	1861	1164	1363
Z-statistics	−4.45760***	6.50604***	−5.89439***	9.01233***
Negative return freq.	0.66967	0.33033	0.63144	0.36856
Positive return freq.	0.43418	0.56582	0.38078	0.61922

Note: The equation for z-statistics is $z_i = \frac{\bar{x}_i}{\sigma_i/\sqrt{n_i}}$ for $i \in 1, 2$, where $\bar{x}_i$ is the mean of return in state i, σ is the standard deviation of state i, and n_i is the sample size of state i. *** significant at 1%.

0.6. In short, the results obtained from PSO-HMM are reasonable and more accurate in identifying market states.

4.3. Further explanation to the superiority of PSO-HMM

Intuitively, bull markets have higher yield than bear markets. Based on the bull and bear market states identified by HMM and PSO-HMM, we compare the cumulative returns of R_{t+1} under the two states shown in Fig. 9.

Fig. 9 shows that the cumulative return in state 1 is increasing and the cumulative return in state 0 is decreasing, indicating that state changes are more likely to continue in the same state, rather than transition to another state (Zhang et al., 2019). In this way, we show that it is possible to profit by identifying market states and get better hedging returns (or avoid unnecessary hedging losses) according to a state-dependent hedging strategy. Furthermore, the return identified based on PSO-HMM is higher than that of HMM in the bull market, whereas in the bear market, the loss of PSO-HMM is greater than that of HMM. The results also imply that the proposed PSO-HMM is more accurate than HMM in state identification.

4.4. Hedging results

In order to effectively compare the hedging effects between different strategies, we selected several common evaluation criteria. The existing literature lists some criteria for measuring hedging effects (Liu, 2014; Zhao et al., 2019), of which the most widely used are the mean of returns, the standard deviation (or variance) of returns, and the ratio of the mean to the standard deviation of returns (Howard & D'Antonio, 1984). We compare these criteria for the hedged portfolio between the model-driven hedge strategy and the market state-dependent hedge strategy. Fig. 10 shows the short process we used to obtain our results.

We set the data from August 1, 2011, to July 31, 2019, as the training set and August 1, 2019, to August 1, 2021, as the test set. Following the forecasting process shown in Fig. 3, we obtain the hedging positions and market states in the testing set with a one-step-ahead sliding forecast. Following the implementation process in Fig. 10, we compare the out-of-sample results of different hedging strategies in Table 3. We also calculate the cumulative returns of different hedging strategies (see Fig. 11).

In Table 3, the mean return is significantly higher with a state-dependent strategy than with a model-driven strategy

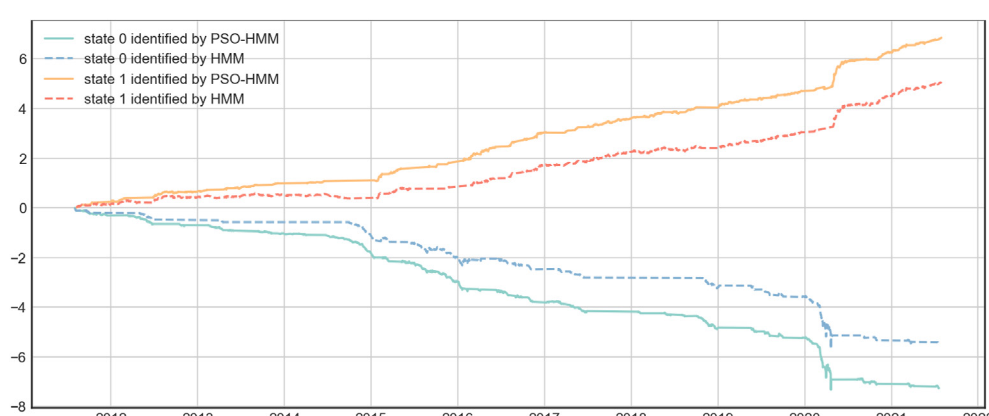


Fig. 9. The cumulative logarithmic return of different states.

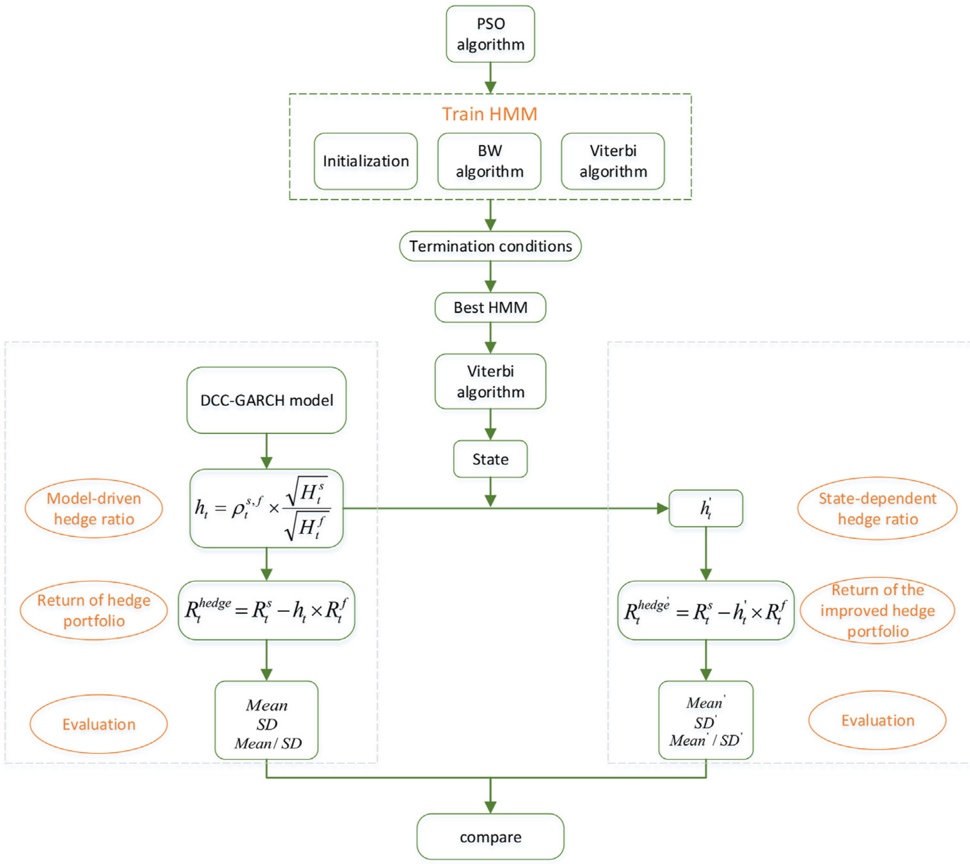


Fig. 10. The process used in obtaining our results.

Table 3
The hedging effect over the full sample period.

	Mean	Standard deviation	Mean/standard deviation
No hedging	0.000378	0.053021	0.001641
Model driven	−0.000609	0.031916	−0.003406
State dependent	0.001598	0.030597	0.009137

with no hedge. The standard deviation differs little between the two strategies but is significantly lower than with no hedging. The ratio of the mean to the standard deviation is far higher with the state-dependent strategy than with the model-driven strategy and no hedge. Therefore, it is clear that the state-dependent hedging strategy increases the returns on the hedge portfolio without significantly increasing the risk of the standard deviation, so its ratio of the mean to the standard deviation is much higher.

And as shown in Fig. 11, after hedging, when the price falls, the hedged portfolio has much smaller losses than the spot, and the state-dependent hedging portfolio has optimal performance. When spot prices rise, spot cumulative earnings grow rapidly,

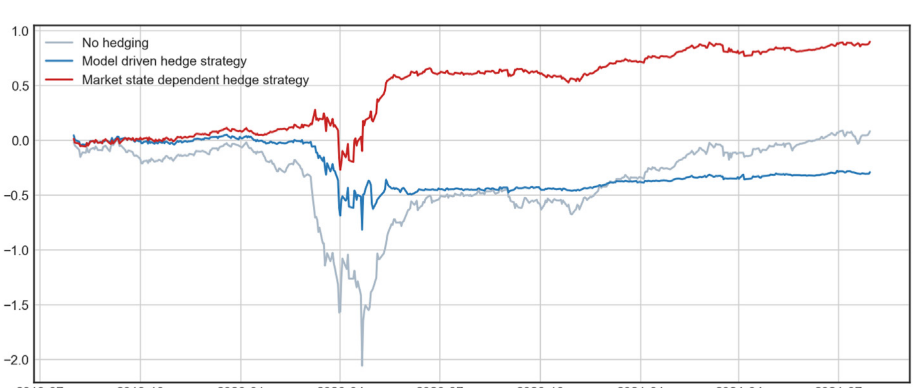


Fig. 11. The cumulative earnings from different hedging strategies.

but the final cumulative earnings are still close to 0 because the losses are very large during price falls. Although the return on the hedged portfolio based on the state-dependent strategy grew rapidly between the end of April and June 2020, when crude oil prices rebounded rapidly, it achieved the highest cumulative returns of the sample period. In sum, we believe that state-dependent hedging strategies are superior to model-driven strategies.

Next we calculate the evaluation criteria every three months, and the results are shown in Fig. 12. The first subplot describes the results of the mean. The state-dependent strategy is mostly superior to the model-driven hedging strategy. Especially in the first half of 2020, although the spot price changed sharply, the state-dependent strategy achieved a positive return that is far higher than that of the model-driven strategy. As shown in the last subplot, we find that the state-dependent strategy has far lower variance than no hedging. Although the state-dependent hedge portfolio has a larger standard deviation than the model-driven portfolio, the minimum standard deviation was achieved in April 2020, indicating that the hedging effect of the model-driven hedging strategy might deteriorate when the price changes very sharply, whereas the state-dependent portfolio performs well. Based on the ratio of the mean to the standard deviation, as shown in the third subplot, the state-dependent hedging strategy is almost the best. In sum, the state-dependent hedging strategy has a good hedging effect in the short term, especially when the price changes sharply.

We also split the sample by two other segmentation points to test the stability of the models over time, and the state-dependent strategy still performs better. The results are shown in Appendix C.

5. Robustness checks

5.1. Different evaluation spans

Because different hedgers have different hedging periods, short or long, in order to test the performance of the state-dependent hedging strategy in different spans, we calculate the hedging results in terms of one, six, and nine months. The results are shown in Table 4. During the one-month evaluation period, the state-dependent strategy had a higher return than the model-driven strategy in 19 of the 24 months and a 17-month return-variance ratio that was higher than that of the model-driven strategy. During the six- and nine-month evaluation periods, the mean and the ratio of the mean to the standard deviation of the state-dependent hedging strategy were higher than those of the model-driven strategy. We find that, for hedgers with different evaluation periods, the state-dependent hedging strategy is desirable, especially for the pursuit of income. Similarly, the proposed model performs well when the market fluctuates violently. Based on a vertical comparison, we conclude that the state-dependent strategy has more significant advantages for long-term hedgers.

The impact of the sharp decline in crude oil prices in April 2020 had a very adverse impact on the market, during which futures and spot prices not only fluctuate sharply but also have a large basis, which poses a great challenge for crude oil hedging. In order to test the hedging results of a state-dependent strategy under extreme market conditions, we select this period to evaluate the hedging effect. In the penultimate row in 4, during the oil shock period, the robustness of the market state forecast is reflected, and the state-dependent strategy still has good performance in response to extreme market changes.

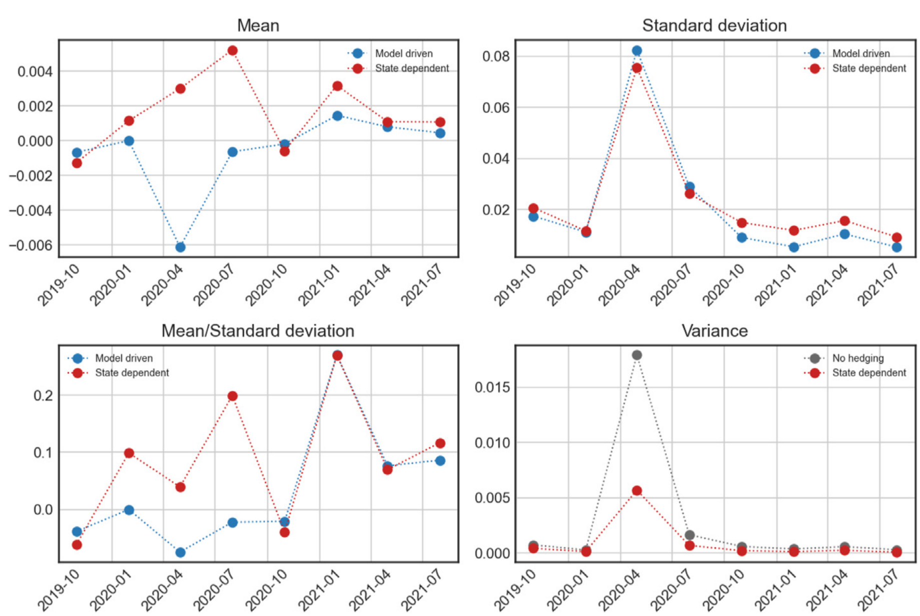


Fig. 12. Hedging effect evaluated every three months.

Table 4
Hedging performance for different evaluation periods.

Period	Mean		Standard deviation		Mean/Sstandard deviation		Variance		
	Model-driven	State-dependent	Model-driven	State-dependent	Model-driven	State-dependent	No hedging	State-dependent	HE
Panel I. Hedging performance for the evaluation period of 1 months									
2019–08	0.000211	**0.000468**	0.019994	**0.017533**	0.010546	**0.026673**	0.000670	0.000307	**0.541179**
2019–09	−0.001132	−0.004076	0.018207	0.028808	−0.062169	−0.141490	0.001329	0.000830	**0.375343**
2019–10	−0.001095	**−0.000456**	0.014630	0.014938	−0.074844	**−0.030520**	0.000359	0.000223	**0.377653**
2019–11	0.002027	**0.002574**	0.010131	**0.009803**	0.200091	**0.262592**	0.000158	0.000096	**0.392471**
2019–12	−0.000637	**0.001113**	0.009463	0.011836	−0.067269	**0.094019**	0.000224	0.000140	**0.373501**
2020–01	−0.001505	**−0.000306**	0.013585	**0.013401**	−0.110793	**−0.022833**	0.000396	0.000180	**0.546316**
2020–02	0.000087	**0.003137**	0.011469	0.014193	0.007616	**0.221012**	0.000671	0.000201	**0.699999**
2020–03	−0.016313	**−0.002362**	0.049067	0.061524	−0.332472	**−0.038384**	0.007884	0.003785	**0.519877**
2020–04	−0.002118	**0.008217**	0.135539	**0.116725**	−0.015624	**0.070393**	0.045770	0.013625	**0.702323**
2020–05	−0.001249	**0.014378**	0.050039	**0.041659**	−0.024970	**0.345126**	0.003304	0.001735	**0.474795**
2020–06	−0.000962	**0.001764**	0.011933	0.015405	−0.080622	**0.114493**	0.001272	0.000237	**0.813489**
2020–07	0.000226	0.000011	0.006624	0.009011	0.034130	0.001186	0.000208	0.000081	**0.608775**
2020–08	0.000833	**0.001626**	0.008548	0.011364	0.097471	**0.143101**	0.000230	0.000129	**0.439151**
2020–09	0.000071	−0.001008	0.011556	0.019491	0.006127	−0.051694	0.000851	0.000380	**0.553495**
2020–10	−0.001345	−0.002226	0.006916	0.013246	−0.194494	**−0.168071**	0.000731	0.000175	**0.759936**
2020–11	0.002416	**0.005129**	0.005694	0.015613	0.424360	0.328480	0.000618	0.000244	**0.605429**
2020–12	0.001478	**0.002221**	0.005758	0.008662	0.256731	0.256362	0.000185	0.000075	**0.594089**
2021–01	0.000523	**0.002289**	0.004804	0.010593	0.108939	**0.216097**	0.000295	0.000112	**0.619684**
2021–02	0.001710	**0.005246**	0.005272	0.008561	0.324300	**0.612827**	0.000182	0.000073	**0.598021**
2021–03	−0.000571	−0.003074	0.008545	0.019650	−0.066768	−0.156416	0.001234	0.000386	**0.687139**
2021–04	0.001364	**0.001501**	0.015197	0.015804	0.089783	**0.094987**	0.000289	0.000250	**0.135567**
2021–05	−0.000015	−0.000810	0.004675	0.009112	−0.003296	−0.088890	0.000293	0.000083	**0.716175**
2021–06	0.001274	**0.002619**	0.003475	0.005647	0.366558	**0.463695**	0.000080	0.000032	**0.599896**
2021–07	0.000143	**0.001394**	0.007036	0.011701	0.020380	**0.119098**	0.000522	0.000137	**0.737864**
Panel II. Hedging performance for the evaluation period of 6 months									
2020–01	−0.000335	**−0.000072**	0.014618	0.016817	−0.022941	**−0.004300**	0.000508	0.000283	**0.443330**
2020–07	−0.003332	**0.004125**	0.061247	**0.055941**	−0.054398	**0.073744**	0.009849	0.003129	**0.682252**
2021–01	0.000617	**0.001253**	0.007507	0.013554	0.082232	**0.092406**	0.000505	0.000184	**0.636339**
2021–07	0.000625	**0.001085**	0.008199	0.012756	0.076245	**0.085051**	0.000437	0.000163	**0.627381**
Panel III. Hedging performance for the evaluation period of 9 months									
2020–04	−0.002242	**0.000940**	0.048670	**0.045290**	−0.046056	**0.020759**	0.006261	0.002051	**0.672379**
2021–01	0.000194	**0.002595**	0.017884	0.018827	0.010823	**0.137843**	0.000919	0.000354	**0.614249**
2021–10	0.000625	**0.001085**	0.008199	0.012756	0.076245	**0.085051**	0.000437	0.000163	**0.627381**
Panel IV. Hedging performance during the period of the oil shock									
Oil shock period 1	0.001769	**0.026258**	0.137414	**0.101527**	0.004772	**0.082409**	0.046589	0.010308	**0.778752**
Oil shock period 2	0.004005	**0.020763**	0.114086	**0.093376**	0.011859	**0.067947**	0.034717	0.008719	**0.748854**
Oil shock period 3	−0.002885	**0.012219**	0.102839	**0.089478**	−0.008997	**0.040847**	0.024777	0.008006	**0.676857**

Note: The column labeled HE represents the hedging efficiency of the model-driven strategy. HE can be calculated by $HE = 1 - \frac{var(R_{hedge})}{var(R_s)}$, $var(R_{hedge})$ and $var(R_s)$ are the variance of the hedging portfolio's return and the variance of spot's return, respectively. Oil shock periods 1, 2, and 3 are April 10–30, 2020; April 1-May 10, 2020, and March 20-May 20, 2020, respectively.

5.2. Different adjustment range in model-driven hedge ratio

A characteristic of the state-dependent strategy is to adjust the positions obtained by the traditional volatility models according to the market state. When the market state is positive, a hedger is recommended to reduce model-driven positions to pursue spot returns. Likewise, when market conditions are negative, a hedger can obtain additional returns on futures by increasing model-driven positions. In our empirical study, we assume that $\theta_{up} = 0.5$, $\theta_{down} = 1.5$. In this section, we perform a robustness test of the adjustment range. We select three pairs of values for (θ_{up}, θ_{down}) and calculate the results on the three-month evaluation period (see Table 5). Regardless of how (θ_{up}, θ_{down}) is adjusted, we can conclude that the mean and the ratio of the mean to the standard deviation of the state-dependent strategy are higher than those with the model-driven strategy, and the state-dependent hedge strategy is better than no hedging, based on the variance.

We also find that, when θ_{up} becomes smaller and the value of θ_{down} becomes larger, the gap between the two strategies becomes larger by every evaluation criterion. Therefore, the state-dependent strategy yields more benefits and performs better.

5.3. Different data frequency

To compare the performance of the method to hedge oil in the short and long term, we change the frequency of the data. As with the daily hedging strategy, we use weekly and monthly data for one-step sliding state identification and hedging position forecasting to achieve long-term state-dependent hedging.

Table 5
Hedging results with different adjusting ratios.

	Date	2019–10	2020–01	2020–04	2020–07	2020–10	2021–01	2021–04	2021–07
Mean	Model-driven	−0.000664	0.000004	−0.006115	−0.000635	−0.000184	0.001457	0.000798	0.000457
	(0.8, 1.2)	−0.000901	0.000463	−0.002470	0.001707	−0.000346	0.002147	0.000916	0.000705
	(0.8, 1.4)	−0.000991	0.000759	0.000603	0.001463	−0.000186	0.001962	0.000719	0.000675
	(0.6, 1.6)	−0.001228	0.001218	0.004248	0.003804	−0.000348	0.002652	0.000837	0.000923
Standard deviation	Model-driven	0.017423	0.011140	0.082455	0.029025	0.009047	0.005398	0.010430	0.005287
	(0.8, 1.2)	0.017942	0.010956	0.077431	0.025290	0.010631	0.007608	0.011724	0.006116
	(0.8,1.4)	0.019376	0.010854	0.074715	0.025327	0.011756	0.007823	0.012722	0.006814
	(0.6, 1.6)	0.021592	0.011278	0.075247	0.025077	0.014895	0.010635	0.015527	0.009096
Mean/Standard deviation	Model-driven	−0.038131	0.000369	−0.074157	−0.021860	−0.020329	0.269995	0.076542	0.086491
	(0.8, 1.2)	−0.050244	0.042216	−0.031897	0.067490	−0.032500	0.282228	0.078133	0.115320
	(0.8, 1.4)	−0.051153	0.069968	0.008071	0.057762	−0.015828	0.250752	0.056523	0.099111
	(0.6, 1.6)	−0.056885	0.107988	0.056451	0.151703	−0.023340	0.249313	0.053892	0.101518
Variance	No hedging	0.000740	0.000277	0.017930	0.001658	0.000601	0.000367	0.000586	0.000298
	(0.8, 1.2)	0.000322	0.000120	0.005996	0.000640	0.000113	0.000058	0.000137	0.000037
	(0.8, 1.4)	0.000375	0.000118	0.005582	0.000641	0.000138	0.000061	0.000162	0.000046
	(0.6, 1.6)	0.000466	0.000127	0.005662	0.000629	0.000222	0.000113	0.000241	0.000083

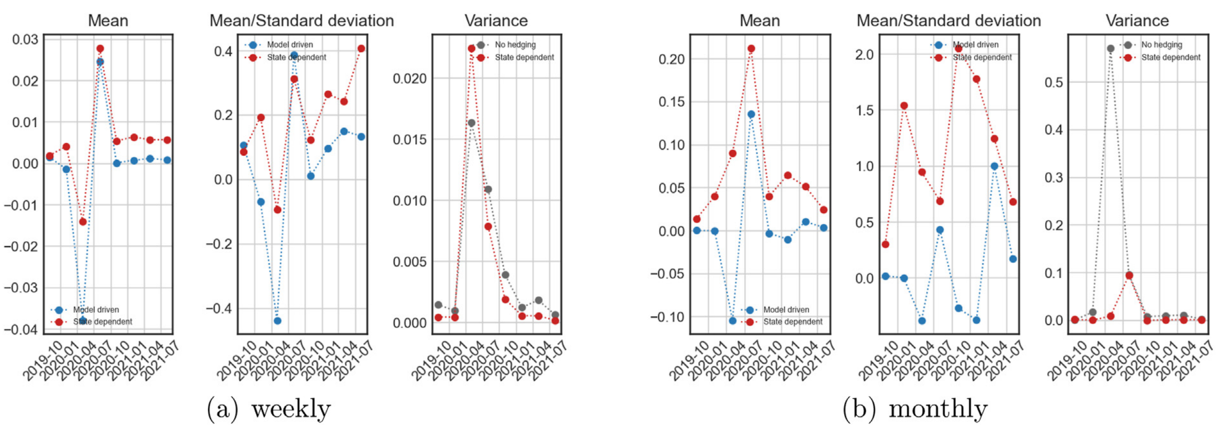


Fig. 13. Hedging results of state-dependent strategy using weekly and monthly data.

Table 6
Robustness test of hedging performance for different benchmark models.

		GARCH	IGARCH	TGARCH	NAGARCH	GJRGARCH
Mean	Model-driven	−0.000609	−0.000612	−0.000356	−0.000502	−0.000624
	State-dependent	0.001598	0.001592	0.001680	0.001565	0.001505
Standard deviation	Model-driven	0.031916	0.031929	0.031865	0.032165	0.032117
	State-dependent	0.030597	0.030537	0.030623	0.030754	0.030451
Mean/Standard deviation	Model-driven	−0.003406	−0.003426	−0.001992	−0.002799	−0.003483
	State-dependent	0.009137	0.009110	0.009598	0.008924	0.008623

We conduct hedging evaluations every three months. In Fig. 13, we find that whether we use the weekly or monthly hedging process, state-dependent strategies achieve a higher mean and ratio of the mean to the standard deviation and greatly reduce risk in the spot market, compared to no hedging. We contend that, in the long run, state-dependent strategies continue to perform well.

Table 7
The hedging effect of WTI crude oil over the full period.

	Mean	Standard deviation	Mean/standard deviation
No hedging	−0.005964	0.154336	−0.015181
Model driven	0.000039	0.059751	0.000158
State dependent	0.010295	0.076497	0.037222

5.4. Different benchmark models of volatility

The model-driven strategies mentioned in this paper refer to the hedge positions obtained based on volatility models, such as GARCH-type models. To test the robustness of the benchmark hedging models, we analyze the hedging results based on different volatility models, such as IGARCH, TGARCH,

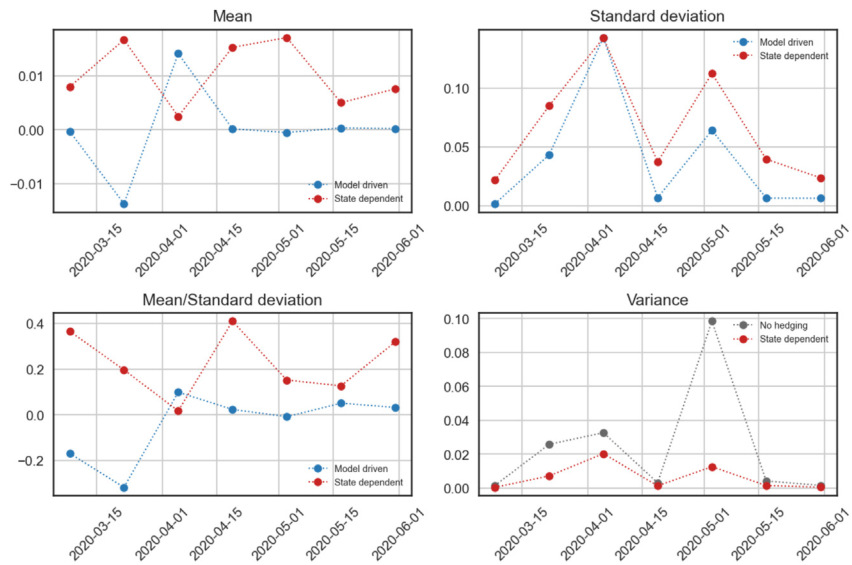


Fig. 14. Hedging effect evaluated every two weeks.

NAGARCH, and GJRGARCH models. The values in Table 6 are the evaluation results in the test set, and the evaluation results for every three months are shown in Appendix D. We find that the state-dependent strategy still has advantages, regardless of the benchmark model used. From this perspective, we propose the use of benchmark-adjusted positions in futures hedging methods.

5.5. Different varieties of crude oil

The WTI crude oil market, as another major global benchmark for crude oil, is used to further verify the hedging ability of the proposed approach. The purpose is to show that our model works not only for Brent risk hedging but also for other crude oil varieties. In addition, WTI crude oil experienced an extreme price decline in 2020 and is a good sample for testing the robustness of the model in extreme cases. In particular, the selection of WTI crude oil better illustrates the importance of risk and return trade-offs because WTI crude oil futures turned negative in April 2020, resulting in significant losses to market participants. In this section, we use data from August 23, 2011, to May 31, 2020. Using February 23, 2020 as the dividing date, the full data period is divided into a training set and a test set. The criteria for measuring hedging effects are shown in Table 7 and Fig. 14. Table 7 shows the hedging effect over the full period, and the results of Fig. 14 are based on every two weeks.

We find that, although the price of WTI rebounded quickly after bottoming out on April 23, 2020, the average return remained negative, even after two months of recovery (see Table 7). It means that investors have a difficult time making up for the losses in the spot market from crude oil in a short time, and hedging with futures to reduce losses is necessary. As seen in Fig. 14, a higher return can be achieved with the state-dependent hedge strategy (see the first subplot in Fig. 14). Although the standard deviation of the hedged portfolio becomes larger, the state-dependent hedge strategy still performs better than the model-driven hedge strategy in a comparison of the ratio of the mean to the standard deviation of returns as a comprehensive evaluation criterion.

Our results lead us to conclude that the hedging strategy based on state identification performs well during sharp price fluctuations. It can also help market participants predict price fluctuations and avoid risk effectively.

6. Conclusion

This paper has two goals. First, to identify market states more accurately in the crude oil markets, we improve the traditional HMM by using the PSO algorithm and propose a new PSO-HHM method. Second, we adjust the model-driven ratio and assess the performance of the proposed hedging model. The main findings from our analysis are as follows.

First, the traditional HMM is easily trapped in local optimization when parameters are estimated. Improper parameter estimation can lead to inaccurate state identification results and even poor decisions. To overcome this problem, we designed a PSO algorithm to obtain globally optimal solutions of initial parameters. The results suggest that the PSO-HMM model is more reasonable and accurate in identifying the market states.

Second, traditionally, dynamic minimum-variance hedge ratios (MVHRs) are determined based on model-driven strategies. However, traditional hedging strategies do not consider the trade-off between avoiding risk and obtaining returns on a hedged portfolio. When market conditions are positive, a hedger who holds a short position in futures can reduce the position driven by the model so as to benefit from the positive market, and the reverse is true as well. The model-driven hedge ratio in this paper is adjusted based on the state of the market. From this point of view, this paper integrates model-driven and market state-dependent hedging strategies to trade off risks and returns. The results show that the state-dependent hedging strategy, whether the evaluation period is long term or short term, has a better hedging effect, and this hedging effect is still amazing when crude oil prices fluctuate significantly. In addition, the hedging results based on weekly and monthly data indicate that a state-dependent hedging strategy can achieve positive returns in long-term hedging.

Third, we test the robustness of the proposed model. The robustness is tested with different hedging evaluation periods, different adjustment ranges of positions, different benchmark models for volatility estimation, different data frequency, different crude categories, and different market conditions, such as the shock in April 2020. The robustness checks on the hedging effect confirm that the proposed model still outperforms the model-driven strategy and provides a better hedging effect than no hedge.

Our study has some limitations, such as it does not consider the presence of transaction costs for hedging. Future research could be conducted, for instance, to detect the presence of transaction costs and to develop other futures hedging models that consider transaction costs and margin calls in the crude oil market, which could produce more realistic and effective results.

Declaration of competing interest

The authors declare that there is no conflict of interest.

Acknowledgments

This paper is supported by Humanities and Social Science Youth Fund project of Ministry of Education of China (Grant No. 21YJC790148); Fundamental Research Funds for the Central Universities (Innovation Funding Projects) (Grant No. 2022CXZZ020); Guangdong Basic and Applied Basic Research Foundation, China (Grant No. 2019B151502037).

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.bir.2022.08.008.

Appendix A. Particle swarm optimization algorithm

Particle swarm optimization is a global search algorithm proposed by Kennedy and Eberhart (1995), which consists of a number of particles flying around in the search space. The PSO requires each individual particle to maintain two vectors during the evolution process: a velocity $v_k = [v_i^1, v_i^2, \cdots, v_i^D]$, a position vector $x_k = [x_i^1, x_i^2, \cdots, x_i^D]$, where the i is the symbol of particle, and D is the dimension of the parameters for a problem. For each particle, there is a best position (pBest) that will be updated when a better position arrives. And the best particle in the swarm symbolized by gBest make the particles converge to the global optimal region. The procedure of the PSO as follows:

Step 1 Randomly initialize a swarm of particles in a D-dimensional search space, then initialize their speed and position, set the individual's historical best pBest as the current position, and the best individual in the group as the current gBest.

Step 2 In each round of evolution, evaluate the fitness of the entire swarm.

Step 3 If the particle's current fitness function value is higher than its previous optimal value, the historical optimal value is replaced with the present position.

Step 4 If the particle's individual optimal is better than the global optimal, the global optimal will be replaced with the particle's optimal value.

Step 5 The speed and position of the dth dimension of each particle i are updated according to the following two equations:

$$v_i^d = \omega \times v_i^d + c_1 \times rand_1^d \times (pBest_i^d - x_i^d) + c_2 \times rand_2^d \times (gBest_i^d - x_i^d)$$

$$x_i^d = x_i^d + v_i^d$$

In the above equation, ω is the inertia weight; c_1 and c_2 are acceleration coefficients (also called learning factors); in the two equations $rand$ is a random number in the interval [0,1].

Step 6 Determine whether the end condition is reached, continuing to step 2 if it does not; otherwise, output gBest and end.

Appendix B. DCC model

Engle (2002)'s DCC model is used to predict dynamic volatility, conditional correlations, and hedging ratios between spot and future returns in this article.

$$R_t^s = \mu_s + \varepsilon_t^s,$$

$$R_t^f = \mu_f + \varepsilon_t^f,$$

$$\left.\begin{pmatrix} \varepsilon_t^s \\ \varepsilon_t^f \end{pmatrix}\right| \psi_{t-1} \sim N(0, H_t),$$

where R_t^s and R_t^f are the day t returns of the spot and the futures, respectively, ψ_{t-1} the set of all the information before day $(t-1)$ and H_t is the conditional covariance matrix modeled as:

$$H_t = \begin{bmatrix} H_t^s & H_t^{s,f} \\ H_t^{s,f} & H_t^s \end{bmatrix} = \begin{bmatrix} \sqrt{H_t^s} & 0 \\ 0 & \sqrt{H_t^s} \end{bmatrix} \times \begin{bmatrix} 1 & \rho_t^{s,f} \\ \rho_t^{s,f} & 1 \end{bmatrix} \times \begin{bmatrix} \sqrt{H_t^s} & 0 \\ 0 & \sqrt{H_t^s} \end{bmatrix} = D_t R_t D_t$$

where R_t is the conditional correlation matrix, and D_t is a diagonal matrix with time-varying standard deviations on the diagonal, $\rho_t^{s,f} = \frac{H_t^{s,f}}{\sqrt{H_t^s H_t^f}}$ is the day t correlation between the spot and the futures returns, H_t^s and H_t^f are the day t variances of the spot and the futures, respectively. The expressions for H_t^f, H_t^f are univariate GARCH class models. Here, only the single-variable GARCH model is described, and other GARCH-type models are detailed by Engle and Bollerslev (1986), Zakoian (1994), Engle and Ng (1993), and Glosten et al. (1993).

$$\begin{cases} H_t^s = \beta_0^s + \beta_1^s \varepsilon_{t-1}^s{}^2 + \beta_2^s H_{t-1}^s{}^2 \\ H_t^f = \beta_0^f + \beta_1^f \varepsilon_{t-1}^f{}^2 + \beta_2^f H_{t-1}^f{}^2 \end{cases}$$

$$R_t = diag\left(\left(Q_t^{-\frac{1}{2}}\right)\right) Q_t diag\left(\left(Q_t^{-\frac{1}{2}}\right)\right)$$

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha z_{t-1} z_{t-1}^{'} + \beta Q_{t-1}$$

Q_t is a symmetric positive definite matrix, $\bar{Q}$ is the unconditional correlation matrix of the spot and the futures returns, and $z_{t-1} = \begin{pmatrix} \varepsilon_t^s/\sqrt{H_t^s} \\ \varepsilon_t^f/\sqrt{H_t^f} \end{pmatrix}$ is the standardized residual vector, The parameters α and β are nonnegative scalars with $\alpha + \beta \leq 1$, and they are related with the exponential smoothing process used to create the dynamic conditional correlations.

Appendix C. The different slide window

We split the sample by two other segmentation points to test the stability of the models over time. The first point is February 1, 2019 (i.e., training sample: August 1, 2011, to January 31, 2019, testing sample: February 1, 2019, to August 1, 2021); the second point is February 1, 2020 (i.e., training sample: August 1, 2011, to January 31, 2020, testing sample: February 1, 2020, to August 1, 2021). The hedged portfolio return based on state-dependent strategy is higher than that based on model-driven strategies (see Tables A and B, Fig. A and B). The hedging strategy based on states is still robust.

Table A: Empirical statistical results, February 1, 2019, to August 1, 2021.

		No hedging	GARCH	IGARCH	TGARCH	NAGARCH	GJRGARCH
Mean	Model-driven	0.000343	−0.000517	−0.000518	−0.000309	−0.000425	−0.000531
	State-dependent		0.000747	0.000747	0.000769	0.000725	0.000644
Standard deviation	Model-driven	0.048144	0.029083	0.029099	0.02904	0.029304	0.029267
	State-dependent		0.028166	0.028112	0.028198	0.028393	0.028041
Mean/Standard deviation	Model-driven	0.001561	−0.003032	−0.003038	−0.001812	−0.002485	−0.003102
	State-dependent		0.00445	0.004454	0.004582	0.0043	0.003846

Fig. A: Hedging results, February 1, 2019, to August 1, 2021.

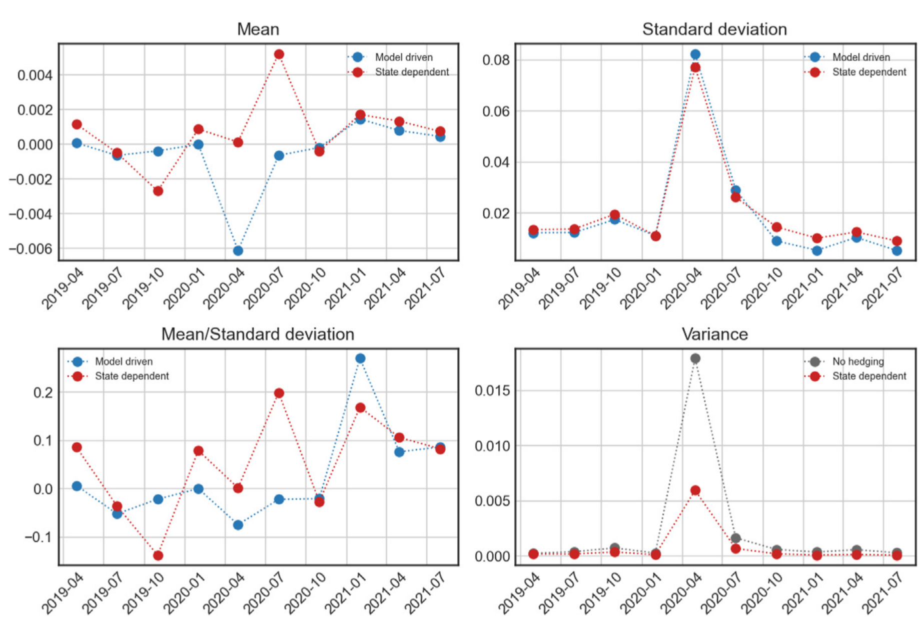
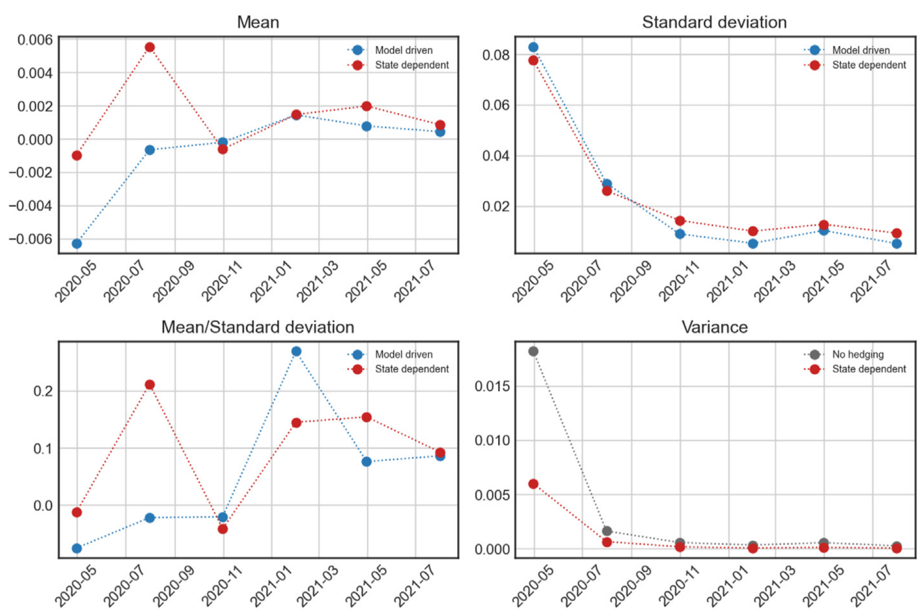


Table B: Empirical statistical results, February 1, 2020, to August 1, 2021.

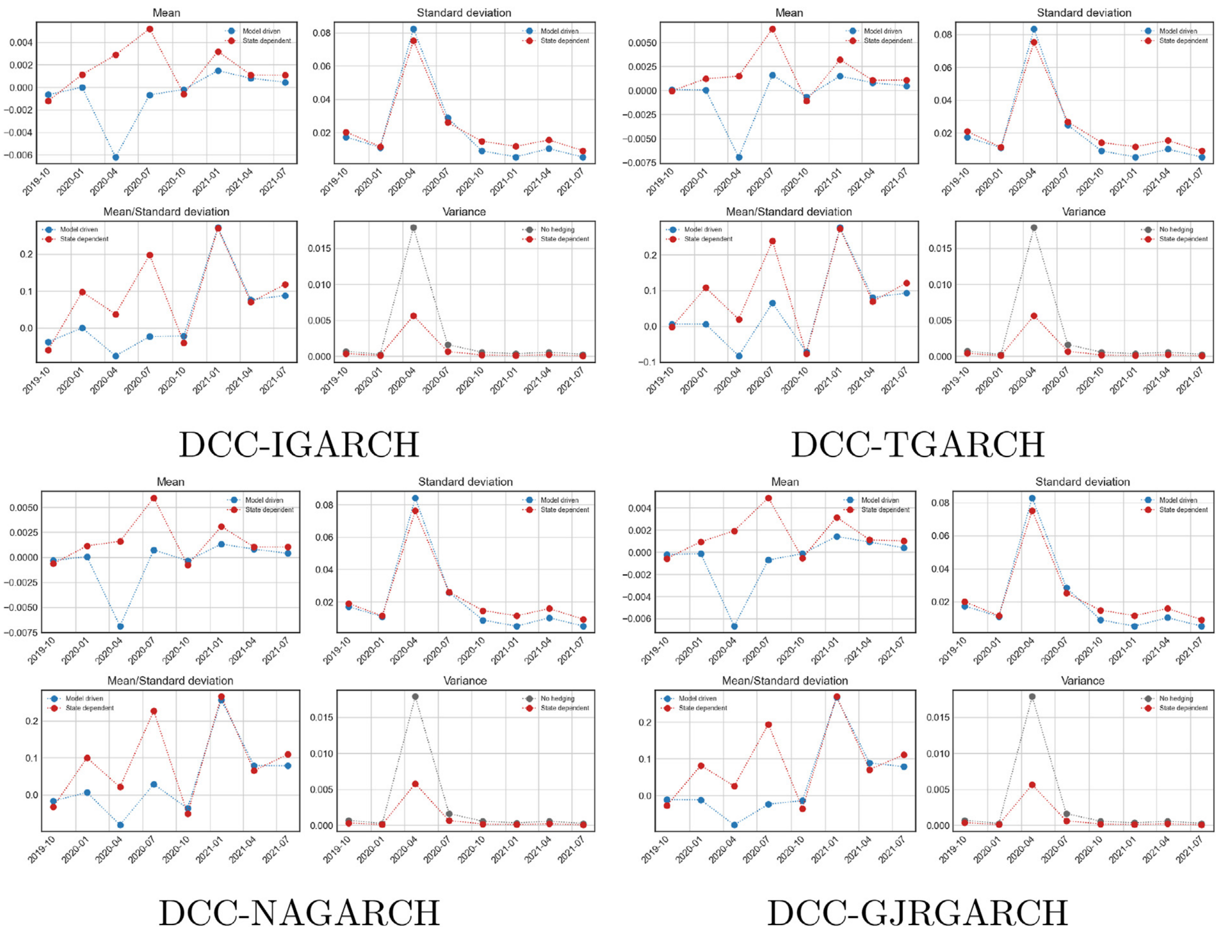
		No hedging	GARCH	IGARCH	TGARCH	NAGARCH	GJRGARCH
Mean	Model-driven	0.000777	−0.000709	−0.000718	−0.000513	−0.000642	−0.000785
	State-dependent		0.001407	0.001392	0.001281	0.001260	0.001207
Standard deviation	Model-driven	0.059925	0.035938	0.035956	0.035869	0.036252	0.036159
	State-dependent		0.034289	0.034244	0.034459	0.034760	0.034250
Mean/Standard deviation	Model-driven	0.003172	−0.003739	−0.003785	−0.002711	−0.003371	−0.004130
	State-dependent		0.007596	0.007520	0.006898	0.006756	0.006520

Fig. B: Hedging results, February 1, 2020, to August 1, 2021.



Appendix D. The different benchmark models

Fig. C: Hedging results under different GARCH models.



References

AlRashidi, M. R., & El-Hawary, M. E. (2009). A survey of particle swarm optimization applications in electric power systems. *IEEE Transactions on Evolutionary Computation, 13*(4), 913–918.

Bae, G. I., Kim, W. C., & Mulvey, J. M. (2014). Dynamic asset allocation for varied financial markets under regime switching framework. *European Journal of Operational Research, 234*(2), 450–458.

Baillie, R. T., & Myers, R. J. (1991). Bivariate Garch estimation of the optimal commodity futures hedge. *Journal of Applied Econometrics, 6*(2), 109–124.

Basher, S. A., & Sadorsky, P. (2016). Hedging emerging market stock prices with oil, gold, VIX, and bonds: A comparison between DCC, ADCC and GO-GARCH. *Energy Economics, 54*, 235–247.

Baum, L. E., Petrie, T., Soules, G., & Weiss, N. (1970). A maximization technique occurring in the statistical analysis of probabilistic functions of Markov chains. *The Annals of Mathematical Statistics, 41*(1), 164–171.

Chang, C., Lai, J., & Chuang, I. (2010). Futures hedging effectiveness under the segmentation of bear/bull energy markets. *Energy Economics, 32*(2), 442–449.

Chkili, W. (2017). Is gold a hedge or safe haven for Islamic stock market movements? A Markov switching approach. *Journal of Multinational Financial Management, 42–43*, 152–163.

Chun, D., Cho, H., & Kim, J. (2019). Crude oil price shocks and hedging performance: A comparison of volatility models. *Energy Economics, 81*, 1132–1147.

Dark, J. (2015). Futures hedging with Markov switching vector error correction FIEGARCH and FIAPARCH. *Journal of Banking & Finance, 61*, S269–S285.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica, 50*(4), 987–1007.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics, 20*(3), 339–350.

Engle, R. F., & Bollerslev, T. (1986). Modelling the persistence of conditional variances. *Econometric Reviews, 5*(1), 1–50.

Engle, R. F., & Ng, V. K. (1993). Measuring and testing the impact of news on volatility. *The Journal of Finance, 48*(5), 1749–1778.

Evrim Mandacı, P., & Kırkpınar, A. (2022). Oil assets and portfolio diversification: Firm-level analysis for Borsa Istanbul. *Borsa Istanbul Review, 22*(3), 571–585.

Glosten, R. L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance, 48*(5), 1779–1801.

Guidolin, M., & Timmermann, A. (2007). Asset allocation under multivariate regime switching. *Journal of Economic Dynamics and Control, 31*(11), 3503–3544.

Hachicha, N., Ghorbel, A., Feki, M. C., Tahi, S., & Dammak, F. A. (2022). Hedging Dow Jones Islamic and conventional emerging market indices with CDS, oil, gold and the VSTOXX: A comparison between DCC, ADCC and GO-GARCH models. *Borsa Istanbul Review, 22*(2), 209–225.

Howard, C. T., & D'Antonio, L. J. (1984). A risk-return measure of hedging effectiveness. *Journal of Financial and Quantitative Analysis, 19*(1), 101–112.

Johnson, L. L. (1960). The theory of hedging and speculation in commodity futures. *The Review of Economic Studies, 27*(3), 139–151.

Kennedy, J., & Eberhart, R. (1995). Particle swarm optimization. *Proceedings – IEEE International Conference on Neural Networks, 4*, 1942–1948.

Kritzman, M., Page, S., & Turkington, D. (2012). Regime shifts: Implications for dynamic strategies. *Financial Analysts Journal, 68*(3), 22–39.

Lalwani, S., Kumar, R., & Gupta, N. (2015). A novel two-level particle swarm optimization approach to train the transformational grammar based hidden Markov models for performing structural alignment of pseudoknotted RNA. *Swarm and Evolutionary Computation, 20*, 58–73.

Liu, W. (2014). Optimal hedge ratio estimation and hedge effectiveness with multivariate skew distributions. *Applied Economics, 46*(12), 1420–1435.

Liu, Z., & Wang, S. (2017). Decoding Chinese stock market returns: Three-state hidden semi-Markov model. *Pacific-Basin Finance Journal, 44*, 127–149.

Maciel, L. (2021). Cryptocurrencies value-at-risk and expected shortfall: Do regime-switching volatility models improve forecasting? *International Journal of Finance & Economics, 26*(3), 4840–4855.

Najkar, N., Razzazi, F., & Sameti, H. (2010). A novel approach to HMM-based speech recognition systems using particle swarm optimization. *Mathematical and Computer Modelling, 52*(11–12), 1910–1920.

Nystrup, P., Hansen, B. W., Larsen, H. O., Madsen, H., & Lindstrm, E. (2017). Dynamic allocation or diversification: A regime-based approach to multiple assets. *Journal of Portfolio Management, 44*(2), 62–73.

Pagan, A. R., & Sossounov, K. A. (2003). A simple framework for analysing bull and bear markets. *Journal of Applied Econometrics, 18*(1), 23–46.

Pan, Z., Wang, Y., & Yang, L. (2014). Hedging crude oil using refined product: A regime switching asymmetric DCC approach. *Energy Economics, 46*, 472–484.

Rabiner, L. R. (1989). A tutorial on hidden Markov models and selected applications in speech recognition. *Proceedings of the IEEE, 77*(2), 257–286.

Rasmussen, T. K., & Krink, T. (2003). Improved Hidden Markov Model training for multiple sequence alignment by a particle swarm optimization-evolutionary algorithm hybrid. *Biosystems, 72*(1–2), 5–17.

Shen, F., Zhao, X., Li, Z., Li, K., & Meng, Z. (2019). A novel ensemble classification model based on neural networks and a classifier optimisation technique for imbalanced credit risk evaluation. *Physica A: Statistical Mechanics and its Applications, 526*, Article 121073.

Shrestha, K., Subramaniam, R., & Rassiah, P. (2017). Pure martingale and joint normality tests for energy futures contracts. *Energy Economics, 63*, 174–184.

Sun, J., Wu, X., Fang, W., Ding, Y., Long, H., & Xu, W. (2012). Multiple sequence alignment using the Hidden Markov Model trained by an improved quantum-behaved particle swarm optimization. *Information Sciences, 182*(1), 93–114.

Viterbi, A. (1967). Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. *IEEE Transactions on Information Theory, 13*(2), 260–269.

Wang, J., Athanasopoulos, G., Hyndman, R. J., & Wang, S. (2018). Crude oil price forecasting based on internet concern using an extreme learning machine. *International Journal of Forecasting, 34*(4), 665–677.

Wang, G., Xie, C., He, L., & Chen, S. (2014). Detrended minimum-variance hedge ratio: A new method for hedge ratio at different time scales. *Physica A: Statistical Mechanics and its Applications, 405*, 70–79.

Yan, Z., & Li, S. (2018). Hedge ratio on Markov regime-switching diagonal Bekk-Garch model. *Finance Research Letters, 24*, 49–55.

Zakoian, J. (1994). Threshold heteroskedastic models. *Journal of Economic Dynamics and Control, 18*(5), 931–955.

Zhang, M., Jiang, X., Fang, Z., Zeng, Y., & Xu, K. (2019). High-order Hidden Markov Model for trend prediction in financial time series. *Physica A: Statistical Mechanics and Its Applications, 517*, 1–12.

Zhang, Y., & Wu, Y. (2019). The time-varying spillover effect between WTI crude oil futures returns and hedge funds. *International Review of Economics & Finance, 61*, 156–169.

Zhao, L., Meng, Y., Zhang, Y., & Li, Y. (2019). The optimal hedge strategy of crude oil spot and futures markets: Evidence from a novel method. *International Journal of Finance & Economics, 24*(1), 186–203.